

# NOTES ON THE NATURE OF SO-CALLED INTRINSIC SYMMETRIES.

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## Abstract

We show that in some cases intrinsic symmetries are peculiar conversion or reflection of external symmetries. They form the structure of the microparticles and then determine interactions of these particles.

**Key-words:** groups, irreducible representations, discrete symmetries, spin, wave equations.

## Introduction

The notion "intrinsic property" has the dual sense. First of all it express some inalienable property, which is inherent in object in any case. But sometimes it reflects a presence of a hidden property or insufficiently known one. Every manifestation of intrinsic or any other property is result of interaction with external objects. Therefore intrinsic properties are relative notions.

In this article exhaustive analysis of some mathematical objects is represented. All they have physical sense such as the electron spin, lepton wave equations. Exhaustive analysis means so full consideration, that it left no possibilities for a continuation of the mathematical analysis and therefore it eliminates the presence of some additional physical characteristics or interpretations. Intrinsic symmetries in below considered examples are the peculiar conversion or reflection of the space-time symmetries and discrete ones.

## 1 Pauli group and spin of the electron

The problem of the spin nature can not be considered solved in spite of a long history of spin concept [1] and its successful mathematical formalization [2] for the electron. At first [1] electron was called "spinning". Then Pauli [2] named it "magnetic". A question on rotation of point like particle is beyond reasonable understanding, therefore spin properties were referred to internal or proper characteristics of the particle. Evidently, the physical picture does not become more understandable.

For a start let us appeal to the generally accepted assumption made by Pauli that the spin is the proper angular momentum of the electron, having the quantum nature and not related to motion of the particle as a whole. This definition has not changed at present. In the case of the electron the proper angular momentum is known to be described by Pauli  $\sigma$ -matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

It is easily checked that  $\sigma$ -matrices generate the 16-th order group just Pauli group [3]. We will denote this group as  $d_\gamma$ . The group has ten conjugate classes. The center of the group contains four elements. The group has eight one-dimensional and two non-equivalent two-dimensional irreducible representations (IR's). The rank of the group is equal to 3. This means that all three Pauli  $\sigma$ -matrices are necessary to generate the group.

Let us introduce the following notations [4]:

$$\sigma_z \sigma_y \equiv a_1, \quad \sigma_x \sigma_z \equiv a_2, \quad \sigma_y \sigma_x \equiv a_3$$

and

$$\sigma_x \equiv b_1, \quad \sigma_y \equiv b_2, \quad \sigma_z \equiv b_3$$

It can be shown that

$$b_1 = a_1 c, \quad b_2 = a_2 c, \quad b_3 = a_3 c, \quad (1)$$

where  $c$  is one of four  $(I, -I, iI, -iI)$  elements of the group center and  $c = \sigma_x \sigma_y \sigma_z = iI$ . Here  $I$  is the unit  $2 \times 2$  matrix. This means that operators  $a_1, a_2, a_3$  are connected with operators  $b_1, b_2, b_3$  by simple relations for the given irreducible representation

$$b_1 = ia_1, \quad b_2 = ia_2, \quad b_3 = ia_3. \quad (2)$$

It can also be noted that

$$a_2 a_1 a_2^{-1} = a_1^{-1} = a_1^3, \quad a_1 a_2 \equiv a_3, \quad a_1^2 = a_2^2 = a_3^2. \quad (3)$$

This means that elements  $a_1, a_2$  generate the quaternion subgroup [5]. Let us denote it as -  $Q_2[a_1, a_2]$ .

Assuming that elements of the group  $d_\gamma$  are generators of some algebra, we obtain the following commutation relations (CR) for the elements of the algebra (infinitesimal operators of the proper Lorentz group representation)

$$\begin{aligned} [a_1, a_2] &= 2a_3, & [a_2, a_3] &= 2a_1, & [a_3, a_1] &= 2a_2, \\ [b_1, b_2] &= -2a_3, & [b_2, b_3] &= -2a_1, & [b_3, b_1] &= -2a_2, \\ [a_1, b_1] &= 0, & [a_2, b_2] &= 0, & [a_3, b_3] &= 0, \\ [a_1, b_2] &= 2b_3, & [a_1, b_3] &= -2b_2, \\ [a_2, b_3] &= 2b_1, & [a_2, b_1] &= -2b_3, \\ [a_3, b_1] &= 2b_2, & [a_3, b_2] &= -2b_1. \end{aligned} \quad (4)$$

The obtained commutative relations coincide with commutative relations of the infinitesimal matrices of the proper homogeneous Lorentz group [6] to the factor 2 common for all equalities. Due to construction of commutative relation (4), all six operators  $a_1, a_2, a_3$  and  $b_1, b_2, b_3$  have a definite physical meaning.

It follows from the first row of commutative relations (CR's) (4) that elements  $a_1, a_2, a_3$  and all their products form the subgroup of 3-dimensional rotations. As it follows from the derivation of commutative relations [6],  $b_1 = \sigma_x$ ,  $b_2 = \sigma_y$ ,  $b_3 = \sigma_z$  have the sense of infinitesimal operators of Lorentz transformations.

Taking into account the anticommutation of the operators  $b_1, b_2, b_3$ , the second upper row of commutative relations (4) takes the form:

$$b_1 b_2 = -a_3, \quad b_2 b_3 = -a_1, \quad b_3 b_1 = -a_2, \quad (5)$$

All three equalities express in infinitesimal form the rotation by some fixed angle of one inertial system with respect to another at their relativistic motion [7]. Obviously, upon deviation from uniform rectilinear motion, this effect has a more complex nature. Upon transition to regular repeated motion, for example, to orbital motion, the rotation also becomes regular, i.e., is manifested as rotation. That is why only the sum of orbital momentum and the spin can be the integral of motion of the particle moving along orbit, rather than orbital momentum and the spin separately.

Thus the analysis of  $\sigma$ -matrix group on the base of CR's (4) which are the direct corollary of the Lorentz transformations demonstrates that so-called proper momentum of the particle with the spin equal  $1/2$  is the consequence of a definite character of motion of this particle, which is not free or rectilinear. This conclusion is in agreement with the well known fact. It is impossible to measure magnetic moment of the electron related with spin momentum, if it moves freely [8].

The explicit form of the operators  $a_1, a_2, a_3$  and  $b_1, b_2, b_3$  for irreducible representations allows to evaluate two weight numbers  $(l_0, l_1)$ , which specify uniquely irreducible representations of the Lorentz group. Calculation of the eigenvalue for the standard  $\sigma$ -matrices yields  $l_0 = 1/2$ . We see, that first weight number  $(l_0)$  coincides with spin value.

The value of the first weight number is determined formally by operators  $a_1, a_2, a_3$ , i.e. by the subgroup of three-dimensional rotations. But generation of the spin rotation is impossible without relativity, as it follows from the above mentioned. It is undoubtedly truly to correlate spin with quite a definite quantum number, if the quantum numbers are interpreted as indices of groups [9]. But it is an unprovable assumption to endow the spin notion with a physical value which exists separately from the motion of the electron as a whole.

Thus two sides of the spin conception has been demonstrated. The first is related to the form of equation. It determines the fermion or boson type of particles. As it is shown above this side stems from the three-dimensional rotation subgroup. This yields the strict fixing of the spin value as the integer or half-integer constant, rather than physical quantity. It is this side of spin notion that is present in the spin definition proposed by Pauli.

The second side is related to the occurrence of the physical quantity of the spin momentum of the electron (and corresponding magnetic momentum) in the interaction resulting in nonuniform motion. If the motion becomes periodical, repeated, we obtain the particle spin as the physical quantity. In this manifestation (according to the second row of commutation relations (4)) spin is no more a strictly fixed constant. The circumstance that the first side related to the form of equation is initial obligatory and independent on the second becomes fundamental. The second side is realized only in the presence of the nonuniform motion and depends on the first

one in its manifestations and details. This can influence significantly the analysis of compound systems or particles with internal structure.

From the generally accepted formalism of the spin equal 1/2 without any additional assumption, we obtain one of the irreducible representation of the Lorentz group and, as a corollary, a physical interpretation of Pauli  $\sigma$ -matrices. Strictly speaking, it is applicable for description of electrons or objects, whose structures are not taken into account.

Consequent and more detailed examination of Pauli group structure ( $d_\gamma$ ) show, that it has duality. It means that apart from subgroup  $Q_2[a_1, a_2]$  it contains one more subgroup of eight order -  $q_2[a_1, a'_2]$ . Defining relations between the generators are the same for both groups. Difference is the order of generators. Both generators of  $Q_2[a_1, a_2]$  has fourth order. One generator( $a_1$ ) of  $q_2[a_1, a'_2]$  has fourth order and another is of second one. Commutation relation for  $q_2[a_1, a'_2]$  (Lie algebra) has the form:

$$[a_1, a'_2] = 2a'_3, \quad [a'_2, a'_3] = -2a_1, \quad [a'_3, a_1] = 2a'_2, \quad (6)$$

where  $a_1 = \sigma_z \sigma_y$ ,  $a'_2 = a_2 c$ ,  $a_3 = a_1 a'_2$ ,  $c = \sigma_x \sigma_y \sigma_z = iI$

Let us call  $q_2[a_1, a'_2]$  as a quaternion group of the second kind. It is not difficult show, that  $Q_2[a_1, a_2]$  is related to  $SU(2)$  with  $\det U = 1$  whereas  $q_2[a_1, a'_2]$  is related to  $SU(2)$  with  $\det U = -1$ .

If we extend  $q_2[a_1, a'_2]$  by the same element as previously  $c = \sigma_x \sigma_y \sigma_z = iI$ , we obtain following commutation relations:

$$\begin{aligned} [a_1, a'_2] &= 2a'_3, & [a'_2, a'_3] &= -2a_1, & [a'_3, a_1] &= 2a'_2, \\ [b'_1, b'_2] &= -2a'_3, & [b'_2, b'_3] &= 2a_1, & [b'_3, b'_1] &= -2a'_2, \\ [a_1, b'_1] &= 0, & [a'_2, b'_2] &= 0, & [a'_3, b'_3] &= 0, \\ [a_1, b'_2] &= 2b'_3, & [a_1, b'_3] &= -2b'_2, \\ [a'_2, b'_3] &= -2b'_1, & [a'_2, b'_1] &= -2b'_3, \\ [a'_3, b'_1] &= 2b'_2, & [a'_3, b'_2] &= 2b'_1, \end{aligned} \quad (7)$$

where  $b'_1 = a_1 c$ ,  $b'_2 = a'_2 c$ ,  $b'_3 = a'_3 c$

These relations are differed from those written above (4). We will connect them with group  $f_\gamma$ , taken into account that  $f_\gamma$  and  $d_\gamma$  are isomorphic.

The representation (7) is called (P)-conjugate with respect to  $d_\gamma$ , since distinctions appear at the level of the 3-dimensional rotation subgroup, i.e. at the first row. The transition from (4) to (7) is equivalent to the following change  $a_2 \rightarrow ia'_2$ . Due to the definition of  $a_3$  we obtain  $a_3 \rightarrow ia'_3$ . All further deviation from commutation relations (4) in more lower rows are the consequences of this primary change. In this case the quaternion subgroup  $Q_2[a_1, a_2]$  transforms into  $q_2[a_1, a'_2]$ .

As a result, three spatial directions are not equivalent for  $q_2[a_1, a'_2]$ , or the so-called asymmetry between the left and right is observed. The first weight number is equal to  $l_0 = 1/2$  for  $q_2[a_1, a'_2]$  only for  $a_1$ . The number  $l_0$  is obtained pure imaginary for other two operators ( $a'_2, a'_3$ ), i.e. they have no physical meaning for the three-dimensional rotation subgroup.

Similar non-equivalence is observed also for  $b'_1, b'_2, b'_3$  which is an equivalent of the Pauli  $\sigma$ -matrix P-conjugate representation. They explicit form for  $f_\gamma$  group is

$$b'_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad b'_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad b'_3 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}.$$

It lead provided some conditions to the spin orientation along the particle momentum. Evidently, it is impossibility to add something more to analysis of Pauli group.

## 2 Lepton wave equations

Next examples, confirming previous one, are the different kinds of lepton wave equations. All they are formulated in the frame work unique approach on the base of unified mathematical formalism.

**Algorithm of the stable lepton equations** was obtained by means of exhaustive analysis of Dirac equation [10]. All completeness of information on Dirac equation is determined by matrices  $\gamma_\mu (\mu = 1, 2, 3, 4)$ . They generate group of 32 order [11] later  $D_\gamma(II)$ . Determining relations for this group have the form [12]

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_{\mu\nu}, \quad (\mu, \nu = 1, 2, 3, 4). \quad (8)$$

It was established that  $D_\gamma(II)$  contains two nonisomorphic subgroup of 16 order only. The first is above examined subgroup  $d_\gamma$  and the second was denoted by  $b_\gamma$ . Lie algebra has the form for  $b_\gamma$ :

$$\begin{aligned} [a_1, a_2] &= 2a_3, & [a_2, a_3] &= 2a_1, & [a_3, a_1] &= 2a_2, \\ [b''_1, b''_2] &= 2a_3, & [b''_2, b''_3] &= 2a_1, & [b''_3, b''_1] &= 2a_2, \\ [a_1, b''_1] &= 0, & [a_2, b''_2] &= 0, & [a_3, b''_3] &= 0, \\ [a_1, b''_2] &= 2b''_3, & [a_1, b''_3] &= -2b''_2, \\ [a_2, b''_3] &= 2b''_1, & [a_2, b''_1] &= -2b''_3, \\ [a_3, b''_1] &= 2b''_2, & [a_3, b''_2] &= -2b''_1. \end{aligned} \quad (9)$$

Transition from (4) to (9) is realized by change

$$b_k \rightarrow b''_k = ib_k \quad (k = 1, 2, 3). \quad (10)$$

Subgroup  $b_\gamma$  in contrast to  $f_\gamma$  is T-conjugate connected component with respect to  $d_\gamma$ .

Then was obtained fourth connected component, i.e. (TP)-conjugate connected component with respect to  $d_\gamma$ . This group was designet  $c_\gamma$ . The corresponding Lie algebra has the form:

$$\begin{aligned} [a_1, a'_2] &= 2a'_3, & [a'_2, a'_3] &= -2a_1, & [a'_3, a_1] &= 2a'_2, \\ [b^*_1, b^*_2] &= 2a'_3, & [b^*_2, b^*_3] &= -2a_1, & [b^*_3, b^*_1] &= 2a'_2, \\ [a_1, b^*_1] &= 0, & [a'_2, b^*_2] &= 0, & [a'_3, b^*_3] &= 0, \\ [a_1, b^*_2] &= 2b^*_3, & [a_1, b^*_3] &= -2b^*_2, \\ [a'_2, b^*_3] &= -2b^*_1, & [a'_2, b^*_1] &= -2b^*_3, \\ [a'_3, b^*_1] &= 2b^*_2, & [a'_3, b^*_2] &= 2b^*_1. \end{aligned} \quad (11)$$

These four constituents are complete set for description of any lepton wave equation.

Algorithm of every concrete lepton equation is based on the following general requirements:

1. The equations must be invariant and covariant under proper Lorentz transformations taken into account all four connected components.
2. The equations must be formulated on the base of irreducible representations of the groups determining every lepton equation.
3. Conservation of four-vector of probability current must be fulfilled and fourth component of the current must be positively defined.
4. The lepton spin is supposed equal to  $1/2$ .
5. Every lepton equation must be reduced to Klein-Gordon equation.

The basis for each equation is the appropriate group of  $\gamma$ -matrices. Each group of  $\gamma$ -matrices is produced by four generators. Three of them must anticommute, and the fourth may anticommute or commute with the first three ones. These requirements comprise necessary and sufficient conditions for the formulation of wave equation for a free stable lepton. None of stated requirements can be excluded without breaking the equation, the Dirac equation in particular. The latter served as a prototype making it possible to formulate the algorithm. As a result, it has been found that each equation for stable lepton has its own unique structure.

1. Dirac equation —  $D_\gamma(II)$ :  $d_\gamma, b_\gamma, f_\gamma$ .  
 $\text{In}[D_\gamma(II)] = -1$
2. Equation for doublet of massive neutrinos нейтрино —  $D_\gamma(I)$ :  $d_\gamma, c_\gamma, f_\gamma$ .  
 $\text{In}[D_\gamma(I)] = 1$
3. Equation for quartet of massless neutrinos —  $D_\gamma(III)$ :  $d_\gamma, b_\gamma, c_\gamma, f_\gamma$ .  
 $\text{In}[D_\gamma(III)] = 0$
4. Equation for massless  $T$ -singlet —  $D_\gamma(IV)$ :  $b_\gamma$ .  
 $\text{In}[D_\gamma(IV)] = -1$
5. Equation for massless  $P$ -singlet —  $D_\gamma(V)$ :  $c_\gamma$   
 $\text{In}[D_\gamma(V)] = 1$ .

Here  $d_\gamma, b_\gamma, f_\gamma, c_\gamma$  are subgroups of the appropriate group of  $\gamma$ -matrices on which one of the four connection components of the Lorentz group is realized. It is seen that the structure of the equation enable to differentiate one equation from another. All of the equations have no substructures allowing for a physical interpretation. For this reason they are stable as the electron. What is more, the proposed method allows an assertion that there are no other stable leptons in the context of the assumption

mentioned above. It turn out possible due to theorem on three types of irreducible matrix groups [11]. New kind of wave equation invariant was used which is numerical characteristic taking one of three possible values  $\pm 1, 0$ . This value is denoted here as  $In[D_\gamma]$ . For example, it is equal to  $In[D_\gamma(II)] = -1$  for Dirac equation [13].

**Unstable leptons and their structures.** The problem of the lepton quantum numbers which make them distinguishable is a cardinal problem for the lepton sector. The first necessary step for it solving is formulation free state for every lepton [14]. It has turned out that it is possible to obtain extra lepton equation within the framework of the previous assumption.

The given task is solved by introducing an additional (the fifth) generator to produce a group of  $\gamma$ -matrices. It was found that there are three and only three possible extensions. Each of them is equivalent to the introducing of it own additional quantum numbers. In this case in new groups, there arise substructures allowing for a physical interpretation in terms of stable leptons. Therein lies their main distinction from the previous ones, making them unstable.

The extension of the Dirac  $\gamma$ -matrix group ( $D_\gamma(II)$ ) with the help of one anti-commuting generator  $\Gamma_5$ , such that  $\Gamma_5^2 = I$ , leads to a group  $\Delta_1$  with a structural invariant  $In[\Delta_1] = -1$ . The extension of the same group with the help of a generator, such that  $\Gamma_5^2 = -I$ , yields a group  $\Delta_3$  with a structural invariant  $In[\Delta_3] = 0$ . Finally, the extension of the group of  $\gamma$ -matrices of the doublet neutrino ( $D_\gamma(I)$ ) with the help of  $\Gamma_5'^2 = -I$  results in a group  $\Delta_2$  with an invariant  $In[\Delta_1] = 1$ .

The order all of the three groups is equal to 64, and each of the groups have 32 one-dimension irreducible representations and two nonequivalent four-dimension ones. Besides, each is composed of three and only three subgroups of the 32nd order, which are isomorphic to one of the five indicated cases. But the composition of the subgroups of the 32nd order is unique in each case.

**Group  $\Delta_1$**  has following defined relations

$$\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\delta_{\mu\nu}, \quad (\mu, \nu = 1, 2, 3, 4, 5) \quad (12)$$

It follows from these relations that

$$\Gamma_6 \Gamma_\mu = \Gamma_\mu \Gamma_6, \quad \Gamma_6^2 = I \quad (\mu = 1, 2, 3, 4, 5), \quad (13)$$

where  $\Gamma_6 \equiv \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5$ .

It means that  $\Gamma_6$  is the center of the group. By this is means that, as we go to an irreducible matrix representations, we can write  $\Gamma_6 = \pm I$ . When  $\mu$  and  $\nu$  run through the value 1,2,3,4 we obtain the Dirac group. It can be shown starting from (12) that, in addition to the Dirac subgroup, the  $\delta_1$  contains two and only two subgroups of the 32nd order. As result we arrive at the following structural decomposition:

$$\Delta_1 \{D_\gamma(II), \quad D_\gamma(III), \quad D_\gamma(IV)\} \quad (14)$$

The relation (14), together with the structure invariant  $In[\Delta_1] = -1$ , identifies the group  $\Delta_1$ , i.e., make its physical contents different from the rest.

**The group  $\Delta_3$**  results from the extension of the Dirac group by means of similar defining relations. The only distinction is in the order of the fifth generator  $\Gamma_5$ .

$$\begin{aligned}\Gamma_s \Gamma_t + \Gamma_t \Gamma_s &= 2\delta_{st}, \quad (s, t = 1, 2, 3, 4), \\ \Gamma_s \Gamma_5 + \Gamma_5 \Gamma_s &= 0, \quad (s = 1, 2, 3, 4), \\ \Gamma_5^2 &= -1.\end{aligned}\tag{15}$$

Hence it follows that

$$\Gamma_6 \equiv \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5, \quad \Gamma_6 \Gamma_\mu = \Gamma_\mu \Gamma_6, \quad (\mu = 1, 2, 3, 4, 5).\tag{16}$$

As before,  $\Gamma_6$  is a group center  $\Gamma_6^2 = -I$ . In this case the matrix realization of representation leads to  $\Gamma_6 = \pm iI$ .

The decomposition of the group has changed as follows:

$$\Delta_3\{D_\gamma(II), \quad D_\gamma(I), \quad D_\gamma(III)\},\tag{17}$$

which corresponds to the structural invariant  $In[\Delta_3] = 0$ . The Dirac subgroup is presented as before, but the composition as a whole has changed.

**The group  $\Delta_2$**  is determined by the relations:

$$\begin{aligned}\Gamma_s \Gamma_t + \Gamma_t \Gamma_s &= 2\delta_{st}, \quad (s, t = 1, 2, 3), \\ \Gamma_s \Gamma_4 + \Gamma_4 \Gamma_s &= 0, \quad (s = 1, 2, 3), \\ \Gamma_4^2 &= -1, \\ \Gamma_u \Gamma_5 + \Gamma_5 \Gamma_u &= 0, \quad (u = 1, 2, 3, 4), \\ \Gamma_5^2 &= -I.\end{aligned}\tag{18}$$

It follows that

$$\Gamma_6 \equiv \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5, \quad \Gamma_6 \Gamma_\mu = \Gamma_\mu \Gamma_6 \quad (\mu = 1, 2, 3, 4, 5),\tag{19}$$

$\Gamma_6$  is an element of the group center and  $\Gamma_6^2 = I$ . In the matrix realization, it takes the form  $\Gamma_6 = \pm I$ .

The group decomposition differs from the two previous ones:

$$\Delta_2\{D_\gamma(I), \quad D_\gamma(III), \quad D_\gamma(V)\},\tag{20}$$

as does structure invariant  $In[\Delta_2] = 1$ . Comparison of the  $\Delta_2$  composition with the five above-cited equations for stable leptons shows that the expression (20) contains only neutrino components.

The presence of the Dirac subgroup in the groups  $\Delta_1$  and  $\Delta_3$  implies that the latter are evident candidates for the description of unstable charged leptons.

It should be noted, that irreducible representation of  $\Delta_2$ -group was obtained in pure real form. This is in agreement with above mentioned theorem on irreducible matrix groups [11]. The theorem states in particular that, if  $In[D_\gamma] = 1$ , there exist a nonsingular matrix that transforms all of matrices of the representation into real ones, which is what has been done. Taken into account the content of  $\Delta_2$ -group, one can say, that it is the basis for description of unstable massive neutrino. This situation is entirely repeats the stable massive lepton one.



## Conclusion

The method has a peculiar "stability" with respect to variations of the generator orders. Their orders equal to four or two are restricted by requirement to have the spin equal to  $1/2$ . The extension of any of the five groups to describe stable lepton by using the anticommuting generator result in the three groups  $\Delta_1, \Delta_2, \Delta_3$  and no others. It would be very difficult to understand the structures of the unstable leptons without whole knowledge of the stable leptons.

In all examined cases the simplest constituents are the connection components of the Lorentz group in different combinations and nothing more. This differences are the basis for manifestation various properties, quantum numbers and so on due to interactions.

All examined cases are particular examples. But the method of the analysis completely general, i.e. exhaustive structural analysis. It is create prerequisites to believe that such examples will be multiply.

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